Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Matrix Presentations of Outputs from Dyna-METRIC Type Models		5. TYPE OF REPORT & PERIOD COVERED Final 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Robert E. Hiller, Ph. D. John P. Wilhelm, Col, USAF Kenneth C. Almquist, Maj, USAF		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS HQ Pacific Air Forces AV (315) 449-6325 Operations Analysis Office COM (808) 449-6325 Hickam AFB HI 96853		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS same		12. REPORT DATE 22 April 1983 13. NUMBER OF PAGES 44
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE not applicable

16. DISTRIBUTION STATEMENT (of this Report)

UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Vol I of two volumes

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Logistics	NORS	Pipeline Theory
Supply	Sustainability	Spare Parts
Aircraft Repair	Dyna-METRIC	Computer Model
NMCS	Backorders	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes a comprehensive way of looking at and interpreting the outputs of a Dyna-METRIC type model. Considerable attention is given to both technical and calculational considerations. The report is intended primarily for analysts who are already broadly familiar with the underlying theory and computational approach or for PACAF staff members who have occasion to use the model outputs. This focus may make it difficult for a nontechnical reader to get through. Nevertheless, for those who want to use this report as a primer on how

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE(*When Data Entered*)

20.

to read and understand the model results contained in the matrices, it will suffice. The reader may ignore the technical portions and concentrate just on the ones that describe the outputs and their interpretation, which is easy to do. The calculational framework is only briefly described in the main body of the report, and the more technical portions are relegated to appendices. The input and output data are bound separately for ease of reference so they can be looked at alongside the descriptive text. For those already familiar with Dyna-METRIC concepts, the way to read the matrices will be almost obvious from their layout. The data, however, have been chosen to illustrate many not-so-obvious points and the text brings these out. Those insights are important to developing an understanding or ability to interpret the model outputs. This report is one of a series related to calculating and interpreting the impact of the availability and management of spare parts in tactical fighter squadrons.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE(*When Data Entered*)

OPERATIONS ANALYSIS OFFICE

VOLUME I

MATRIX PRESENTATIONS OF OUTPUTS FROM DYNA-METRIC TYPE MODELS V. I.

APRIL 1983

PACIFIC AIR FORCES

HQ PACIFIC AIR FORCES

VOLUME I

MATRIX PRESENTATIONS OF OUTPUTS FROM DYNA-METRIC TYPE MODELS

22 April 1983

ROBERT E. HILLER, Ph. D.
JOHN P. WILHELM, Col, USAF
KENNETH C. ALMQUIST, Maj, USAF

HQ PACAF/OA
HICKAM AFB, HI 96853

PREFACE

This report describes a comprehensive way of looking at and interpreting the outputs of a Dyna-METRIC type model. Considerable attention is given to both technical and calculational considerations. The report is intended primarily for analysts who are already broadly familiar with the underlying theory and computational approach or for PACAF staff members who have occasion to use the model outputs.

This focus may make it difficult for a nontechnical reader to get through. Nevertheless, for those who want to use this report as a primer on how to read and understand the model results contained in the matrices, it will suffice. The reader may ignore the technical portions and concentrate just on the ones that describe the outputs and their interpretation, which is easy to do. The calculational framework is only briefly described in the main body of the report, and the more technical portions are relegated to appendices.

For ease of reference, this report is published in two volumes: (1) descriptive text and (2) input and output data. For those already familiar with Dyna-METRIC concepts, the way to read the matrices will be almost obvious from their layout. The data, however, have been chosen to illustrate many not-so-obvious points and the text brings these out. Those insights are important in developing an understanding of or ability to interpret the model outputs.

This report is one of a series related to calculating and interpreting the impact of the availability and management of spare parts in tactical fighter squadrons.

TABLE OF CONTENTS

Matrix Presentations of Outputs From
Dyna-METRIC Type Models

I. Overview of the Supply Model

In the body of this note we describe matrix presentations that show outputs from a dynamic demand-repair logistics model of spare parts availability. The model developed at PACAF is called "Vector." An alternate model based on the same mathematical underpinnings has been developed by Rand and given the name "Dyna-METRIC" (DM). A brief explanation follows for those not familiar with the models.

The aircraft spare parts which are the subjects of the models are those that can be removed and replaced without extraordinary trouble and also can be repaired and thus reused. They are referred to as Line Replaceable Units or LRUs. We may also consider the subunits used to repair LRUs, the so-called Shop Replaceable Units or SRUs.

Both Vector and DM are dynamic models because they permit a flying program (i.e., demand for parts) which is a function of time and they can handle "surge" flying.

Consider a single part type designated by a National Stock Number (NSN) and characterized by data giving:

- Average demands per flying hour
- Probability of base repair
- Repair cycle time in base repair
- Probability of intermediate-level repair at, say, PACAF's Consolidated Intermediate Repair Facility (CIRF)
- Administrative and round-trip travel time to the CIRF
- Depot repair cycle time
- Quantity per Aircraft (QPA); the quantity of this part type (NSN) found on any aircraft

The above data permit a characterization of the stochastic demand and repair processes as outlined schematically in figure 1 below.

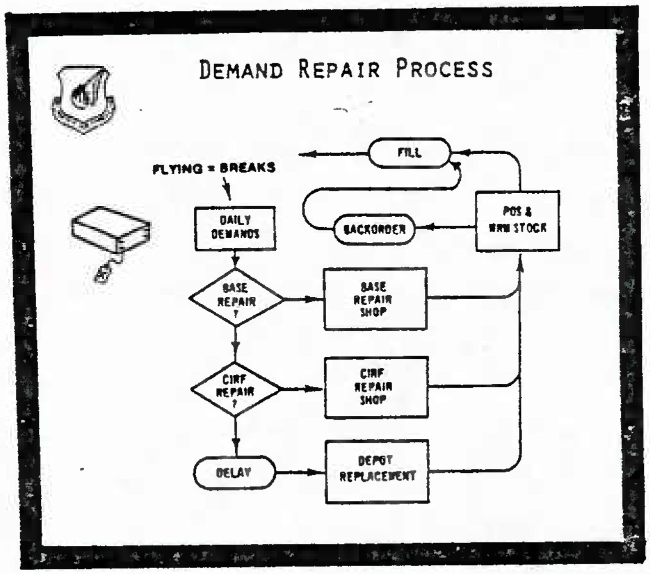


Figure 1

A flying unit is characterized in Vector by the flying program, not essentially by the number of flyable aircraft. The flying program determines, in effect, the spare part demands per unit time of each NSN. As is commonly done, we assume that breaks occur as a linear function of flying hours. This point is arguable, but the consequences are not germane to this report. The other factors determine the time the part spends in one or another of the repair loops.

Although we define the input data in terms of averages, we are very concerned with the random fluctuations that occur in both numbers of

demands and repair loop times. Certain reasonable assumptions are made concerning underlying statistical distributions which permit probability statements to be made about the statistical fluctuations of parts (by NSN) in the repair process. Of the underlying assumptions necessary to the mathematical derivations, the dynamic Palm's Theorem (developed by Crawford at PACAF/OA and by Hillestadt and Carrillo at Rand) says that the number of parts in the repair line have a Poisson distribution and tells us how to calculate the expected value of "parts in repair." These two statements contain all the information that can be wrung from the data. All else is elaboration and interpretation, but that is not a trivial process. This report is really about "how to understand what the calculations mean."

The primary objective of the model is to evaluate a stock asset position against a flying program or perhaps vice versa. This is accomplished by computing the statistical probabilities that the number of parts in repair exceed the number in stock asset for each NSN in the list of important LRUs. Each such occurrence leads to a "hole" in an aircraft and the hole persists until a part returns after being repaired to fill it. Any aircraft with holes is considered to be in Not-Mission-Capable-Supply (NMCS) status.

The data for each and every NSN of interest are run through the computational process to produce probabilities for each of 0, 1, 2, etc, stock shortages, i.e., holes. Using these probabilities, we can then calculate and display for discrete points in time:

- $p_i(k)$, the probability (for the i^{th} NSN) of k shortages
- EBO_i, average shortages for the i^{th} NSN
- SL, additive stock required to make $p_i(0) \geq .99$
- EBO, sum of average shortages over all parts

- P(NMCS=k), the probability that NMCS=k for 100% canning

- P(Holes=k), the probability that total holes = k

Full, or 100% canning referred to above means consolidating holes to the fewest possible number of aircraft. The fewest number of NMCS aircraft under 100% canning is determined by that NSN where number of holes (divided by Quantity Per Aircraft) is largest at the given time. The other extreme would be no cannibalization. In this situation the hole remains where it occurred until a part is returned from repair. A no-cann policy has the potential of creating so many NMCS aircraft that the unit cannot meet its flying program. Both policies are too extreme for normal peacetime operations; the pragmatic policy lies somewhere in between. Nevertheless, the extreme ones are useful boundary cases in analyzing the fleet condition.

II. The Structure and Arithmetics of the "Vector" Model

Introduction

A Dyna-METRIC type model manipulates a large amount of demand and repair information for each member of a set of NSNs. The computational results are ultimately reduced to a highly condensed form: the "expected number in repair at time t for part type i," denoted by $\lambda_i(t)$. Once $\lambda_i(t)$ has been calculated, the entire story is in hand -- we then know what Dyna-METRIC has to say about the input data. Our problem from that point on is merely to interpret the meaning of the $\lambda_i(t)$.

The interpretation of such a "rich" variable, however, is not trivially simple. The present output from RAND's Dyna-METRIC computer model does not show the $\lambda_i(t)$ explicitly. Rather, it hints at them by portraying some of the consequences. RAND unquestionably made such a choice when developing the computer program because the model needed to span a wide range of possible uses at many locations, each with different interests. The choice was indeed reasonable for situations of interest to, say, an Air Logistics Center where calculations would span many different bases. If great detail were provided in the printouts, the amount of paper would be overwhelming.

Here in PACAF, however, our interests usually focus on one base or one flying unit. For that situation, we have come to believe that a total exposure of the $\lambda_i(t)$ leads to a much clearer perception of their consequences and what is happening in a dynamic logistics system than does piecemeal information. Toward that end, we have developed easy-to-comprehend matrix presentations of the $\lambda_i(t)$ and their consequences.

Computationally, the integral form of $\lambda_i(t)$, which is at the heart of time-dependent pipeline calculations, is replaced by a discrete sum in the realization of the theory we call the "Vector" model. Within the accuracy boundaries imposed on us by the available data, the sum is equivalent to the integral. As will be seen, using a discrete summation

permits an easy calculation of $\lambda_i(t)$ and also permits us to look at a variety of demand functions, allied to different sortie-rate regimes, and a variety of repair functions. We also find it easy to explore many different time-dependent stock options.

The Convolution Integral

In the illustrations that follow, we consider only simple repair loops--those in which the many-server assumption is valid and indenture relationships are not considered. Such complexities can be added, as required, in the same way they are treated in Dyna-METRIC.

Given that the input information describing the demand and repair processes are available, the actual calculation of $\lambda_i(t)$ is straightforward. Following the notation of Hillestad and Carrillo (ref 1), $\lambda(t)$ is given on page 9 as a convolution integral:

Eq (1) $$\lambda(t) = \int_0^t m(s)\, \overline{F}(s,t)\, ds$$

where

$\overline{F}(s,t)$ is the probability that a part going into service at time s is still in service at time t. In the calculations we discuss, we will take $\overline{F}(s,t)=\overline{F}(t-s)$, i.e., it will depend only on the time difference. This is not a limitation, for we may still have a different function for each t.

$m(s)$ is the demand intensity function at time s, discussed more fully later.

The integral of eq (1) can be approximated by a sum:

$$\text{Eq (2)} \qquad \lambda(t) = \sum_{s=o}^{s=t} m(s)\, \overline{F}(s,t)$$

where $m(s)$ and $\overline{F}(s,t)$ are written as discrete functions.

In the case at hand, the discrete summed form, although it looks only at daily intervals, is as adequate as our knowledge of the functions $\overline{F}(s,t)$ and $m(s)$.

It is a simple matter to define $\overline{F}(s,t)$ and $m(s)$ as vectors of values for discrete time intervals. The "tabular" definitions of the functions allow full and easy access to all kinds of functions. The vector scalar products of eq (2) are easily computed to obtain values of $\lambda(t)$ for each value of t. The computational process is described further in Appendix B.

The Demand Intensity Function

Under Dyna-METRIC assumptions, demand for the i^{th} part on day s is characterized by a Poisson distribution having expected value $m(s)$. Since we are working with discrete one-day intervals, $m(s)$ is defined by a string of discrete values.

Letting:

D_i = demands per flying hour for the i^{th} part, a constant;

F = flying hours per sortie, a constant;

N = number of aircraft in the flying unit, a constant;

$R(s)$ = daily sortie rate, a function of time; and

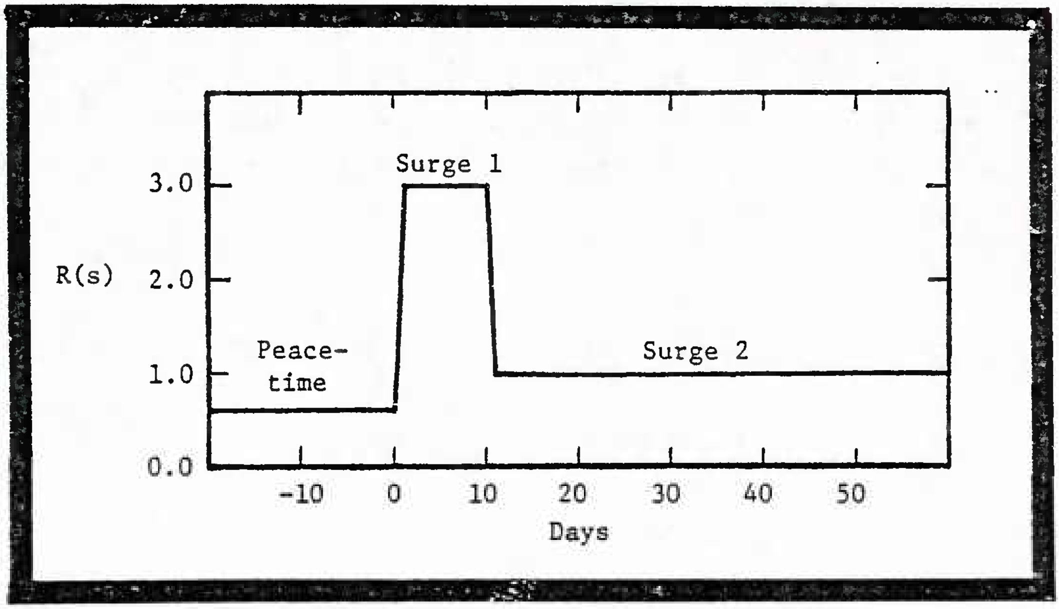
QPA = quantity per aircraft (e.g., an aircraft has 2 landing gears), a constant;

then

Eq (3) $$m(s) = D_i \cdot F \cdot N \cdot R(s) \times QPA$$

R(s) may be any feasible function of time, but in our calculations we typically take it to have one "initial" value appropriate to peacetime training, followed by a "surge" value for a certain number of days, and then it reverts to some "final" value.

Figure 2 Typical Sortie Rate Function



Clearly, the shape of m(s) is determined by the shape of R(s) and its value is determined by the application of the scaling factor $D_i \cdot F \cdot N \cdot QPA$ which depends on the mission of the unit (through F), the size of the unit (through N), and the part in question (through D_i). Actually, all of the factors could be treated as time-dependent; but since it is a trivial extension, it is not set forth explicitly.

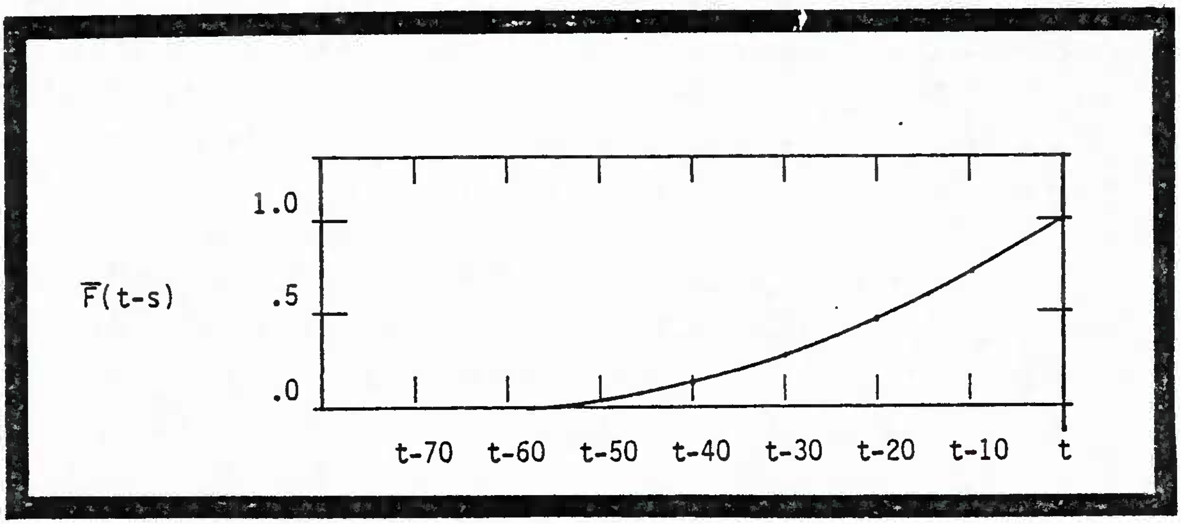
We adopt the convention that day "0" is the end of the "initial" period (the end of "peacetime" rates). Negative indexes then pertain to peacetime flying, positive ones to combat surges. This convention affects the limits of summation of eq (2) in only a trivial way.

The Service Time Distribution

The service-time distribution or repair function, $\overline{F}(t-s)$, can be any form which adequately represents the service process. It is required only that $\overline{F}(t-s)$ satisfy the usual conditions imposed on any distribution function: Namely, it must approach 1.0 as t-s approaches zero, i.e., the part is sure to be unrepaired at the moment it enters repair. Also, $\overline{F}(t-s)$ must approach zero monotonically as (t-s) becomes very large; i.e., after a long enough time, the probability that part is still in service becomes vanishingly small.

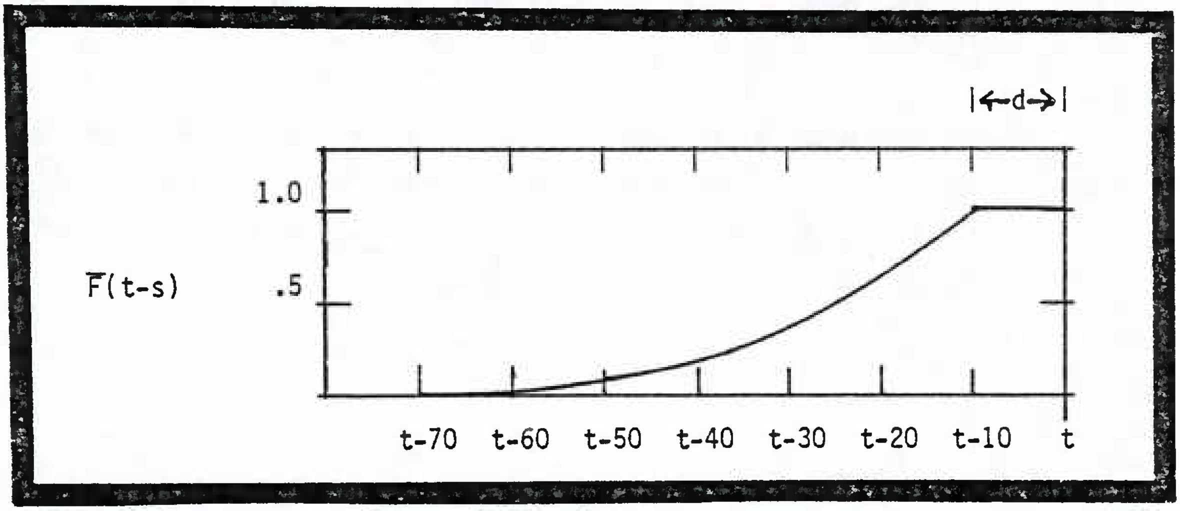
The repair function is defined by a string of numbers which offers the obvious advantage of permitting easy exploration of functions that may not be easy to express in mathematical form. Sensitivity analysis concerning assumed forms of functions thus becomes very easy.

Consequently, $\overline{F}(t-s)$ can be exponential-like:

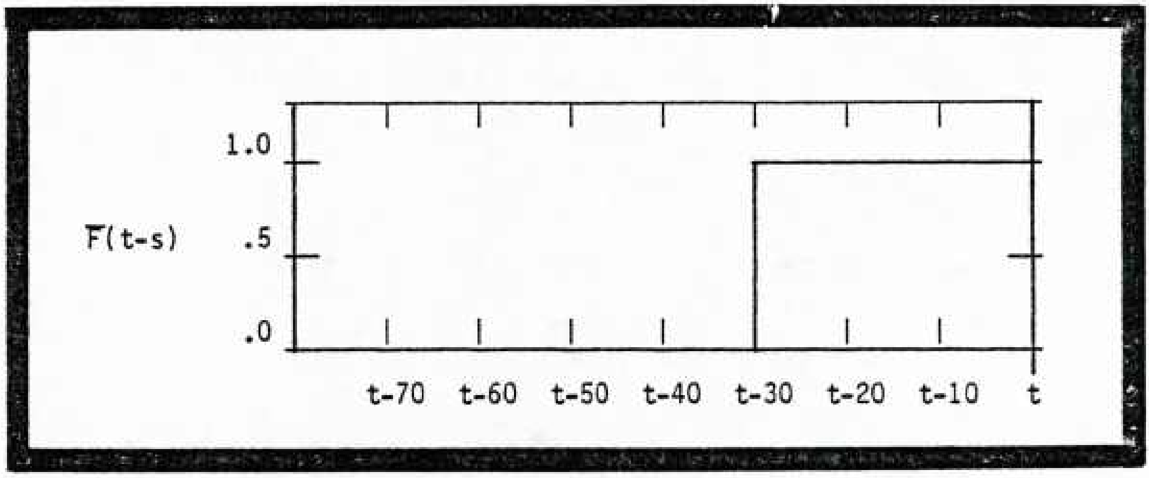


(Note: In portraying these functions, "t" represents now, and "t-s", a time s units in the past, is the point at which a part entered repair.)

Or we can add a constant "admin delay" time, d, before the part starts the repair process:



Or, if we like, we can look at a constant repair time:



For ease of representation, the functions are drawn as continuous but are nevertheless discrete as previously noted.

Clearly, a variety of repair functions can be investigated. In this report, a single form is used for all NSNs (i.e., National Stock Number) but the specific values of the parameters are adjusted by the data characterizing each one. The form used for illustration herein is that of the last graphic, the constant repair time.

Input Data

The data which we use for this report are synthetic and were chosen only for illustration although they are reasonably typical.

The number of parts used for illustration are limited so they will fit easily on a single page. We look only at 42 NSNs, whereas a full set of Line Replaceable Units (LRUs) for a fighter aircraft could ordinarily number several hundred. In actuality, initial computational scans on a set of real-world LRUs could permit us to focus on a small subset of "critical"

items which largely determine the overall system performance. The NSNs in the present illustrative set of 42 could represent such a "critical" group.

The data describing the individual LRUs are estimates of:

- Demands per flying hour
- Not Repaired This Station (NRTS) rate at base
- Repair cycle time at base
- Administrative delay time and round trip shipping time to the Centralized Intermediate Repair Facility (CIRF)
- NRTS rate at CIRF
- Repair cycle time at CIRF
- Order and ship time from depot
- Quantity per application

We also specify:

F = 1.2 hours per sortie

N = 48 aircraft

$$\text{Sortie rate, } R(s) = \begin{cases} 0.6 & \text{for peacetime (per calendar day, not flying day)} \\ 3.0 & \text{for first 10 days of combat} \\ 1.0 & \text{thereafter.} \end{cases}$$

which establishes the flying program as a function of time.

III. Description of Model Inputs and Outputs and Their Interpretation

Description of Stock Assets
(Matrix 1)

Examples of model inputs and outputs (matrices) are contained in companion Volume II of this report and should be examined while reading the text.

Our broad goal in running the Vector model is to make evident how each NSN contributes to the expected NMCS (not mission capable for supply) rate for the fleet.

It is trite to observe that NMCS depends upon the stock level for each NSN, but that observation nevertheless serves to introduce the notion that we must describe the stock asset position before we can calculate NMCS. In our calculations we define the flying unit's stock assets by a column vector which has as entries the number of items stocked for each NSN. We also arrange to store several different stock vectors side by side, each describing a different stock position, for we will want to explore what happens under different stock options. An example is given by Matrix 1.

Individual vectors can be used to represent different situations:

- A zero vector means we have no stock at all.
- The authorized peacetime stock is represented by one vector.
- The actual peacetime stock is given by yet another vector.
- Still another stock option is "Peacetime Operating Stock (POS) plus War Readiness Materiel (WRM) stocks." This is the appropriate one to use when a unit may draw freely from its total stock.

- Such vectors can also be used to represent time changes in stock levels as could occur when a deploying TAC unit brings stock with it. A distinct vector would be associated with each period during which the stock was "constant."

- etc.

Note that "stock," as we use the term, refers to the total asset, not to where it is located. It consists of that which is in the pipeline <u>and</u> that which is "on the shelf," not just one or the other.

Evaluating the Sum

The details of the computations involved in evaluating eq (2) which yield $\lambda(t)$ (page 7) are covered in Appendix B. Several points, however, are worth brief mention here before we look at the model's outputs of $\lambda(t)$.

- There are three repair pipelines in PACAF -- a base repair cycle, a Centralized Intermediate Repair (CIRF) repair cycle, and a depot repair cycle. In which of these pipelines an individual part finds itself depends partly on its characteristics, i.e., some parts may be repaired at the base depot only, some at the CIRF and depot only, etc., and partly on the probabilities that describe repair line performance in accordance with the input data describing the part.

- Accordingly, we will be interested in a $\lambda_i(t)$ for each of three pipelines as well as one for the sum [$\lambda(t)$] of all three pipelines.

- We also must be careful that sufficiently long "tails" have been included in the $\overline{F}(s,t)$ vectors augmented by zeros if required, so that peacetime operations are in "steady state" before looking at the transient behavior. We display $\lambda(t)$ on four peacetime days (-6,-4,-2, and 0) to check that the lengths of the vectors used in evaluating the convolution integral were sufficiently long to guarantee a stationary state, which is evidenced by an unchanging value of $\lambda(t)$.

The Matrix Display of $\lambda(t)$
(Matrices 2 through 5)

The next four printouts display the $\lambda_i(t)$ matrices: one for the base repair loop, one for the CIRF loop, one for the depot loop, and one for the sum of the three basic loops.

The meaning of most of these displays is self-evident from the preceding text, but there are a few areas worth singling out. Some numbers are echoes from the input data base. "QPA" is quantity per aircraft. "Time" is the expected repair time for that NSN in that particular repair loop. "PCT" indicates the probability that the part will be repaired in that loop.

The matrices showing $\lambda_i(t)$ for each of the repair loops may be of some interest if we want to pursue questions relating to repair time of specific NSNs. More often, we are interested in the overall picture which is shown by Matrix 5, the total expected value of part type i in all repair cycles.

At its most primitive level, Matrix 5 gives an indication of the average number of each NSN in the repair cycle as a function of time. The expected values are constant for the unchanging peacetime program through day 0, rise during the period of the big surge through day 10, and usually show recovery via declining pipeline averages thereafter. Parts that go mostly to the depot may not show recovery until after the depot cycle time has transpired.

Since we know from the fundamental theorem that the individual and the summed repair line contents follow Poisson distributions, we also know how to calculate the probabilities of finding specific numbers of parts in repair. This matrix, then, contains all of the basic statistical information even if it doesn't show it all. All else that can be done is to use the $\lambda(t)$ information in various probability calculations which relate the condition of the aircraft fleet to the "expected parts in repair," the stock assets available, and the cannibalization policy in effect.

We now turn to the task of elaborating the implications of the $\lambda(t)$'s.

Probability Of A Stock Outage
(Matrices 6 and 7)

The probability of a stock outage, i.e., a "backorder," for a particular NSN (i.e., part type) is given by the probability that the number in repair exceeds the stock asset level. Such an outage, therefore, generates a NMCS condition if we do not have a part to replace the one removed from the aircraft. The NMCS condition will persist until a part returns from repair (or we cannibalize it from another aircraft).

A caveat is appropriate at this point. We are aware that not every stock outage makes an aircraft go NMCS; it may be partially mission capable even with the part missing. For convenience, however, we use NMCS as a shorthand version of the more accurate form "Not Fully Mission Capable--Supply."

With the information now at hand, it is easy to calculate the probability of an outage for each NSN. The first step is to calculate the probability that there are k parts in repair, for k = 0, 1, 2...etc. This is immediately available since we know the mean value ($\lambda_i(t)$ from Matrix 5) and that numbers in repair are Poisson distributed. A trivial summing process then gives the probability of k-or-fewer parts in repair. The calculated results are printed out in Matrix 6 for day 0. A separate matrix can be produced for any chosen day.

Although Matrix 6 does not yet bring in the stock assets, the pattern is beginning to emerge: Obviously we would like to have sufficient assets to cover the likely levels of parts in repairs. In Matrix 6, for instance, we look at day 0. For NSN-3, the demand and repair time data are such that there is a 0.99 probability that the number in repair is 5 or fewer. If we have five units as our asset level, we will be covered 99% of the time. Indeed, the column labeled "SL" tabulates the stock asset position needed to cover up to the 0.99 level for each part. It is a good first-cut estimate of what we need in stock assets on that day.

The first-time reader may think that 0.99 is a high level of aspiration. In truth, it is not. When there are, say, 300 NSNs, all of which are needed, each must have a high probability of being available when demanded if the fleet is to be in good shape. For instance, if each of the 300 parts had a 0.01 probability of nonavailability given a demand, then three parts on the average would be unavailable. Under a no-cannibalization policy, 3 aircraft would be NMCS. Or if 100% cannibalization were permitted, we would likely be able to consolidate all the shortages into one aircraft and the NMCS would be 1. This will become more graphic shortly. If for some applications, critical values other than 0.99 are needed, it may be easily changed.

Matrix 7 adjusts Matrix 6 to account for a designated stock option contained in the Stock Option Matrix. It gives the probability of k-or-fewer backorders for the designated stock option.

A "backorder" is defined as a part entering repair for which there is no covering stock. A backorder implies a stock outage and vice versa.

The probability of k-or-fewer backorders is obtained from Matrix 6 merely by shifting each row to the left a number of elements equal to the associated stock asset. The column labeled "SL" is redefined here as being the stock to be added to the asset position to raise the probability of no outage to the 0.99 level. "SL" functions here as a shortage indicator.

The rightmost entry for each NSN shows the expected back orders (EBO) for that NSN and stock option. It, too, is an occasionally useful measure of the adequacy of the stock option.

Three additional lines have been labeled and added at the bottom of the page:

Pr(NMCS.LE.K), the probability that NMCS is less than or equal to k;

PR(NMCS=k), the probability that NMCS is exactly equal to k; and

AVG NMCS, the expected value of NMCS.

These are computed for a policy of 100% cannibalization, i.e., all holes are consolidated into the fewest possible number of aircraft.

For 100% cannibalization, Pr(NMCS.LE.K) is given by the product of the values in the column above it. (Note: If QPA is not equal to 1, the factor in the product is taken from column k = [(k-1)xQPA + 1] for k > 0. The column product will be close to unity only when the value of each entry is close to unity. Merely by scanning the column, the "troublemakers" stand out as those which are significantly less than 1.0. Visually, they are the ones that extend well to the right on each line.

The Pr(NMCS=k) is obtained from the difference of two successive entries:

Eq (4) $\qquad Pr(NMCS=k) = Pr(NMCS.LE.k) - Pr(NMCS.LE.k-1)$

The average NMCS is k times Pr(NMCS=k) summed over all $k \geq 0$.

The distributions of NMCS in either form are quite useful in helping us visualize the spread around either the average (5.35) or most probable (5.0) NMCS.

Finally, the last entry on the page is "Expected Backorders," which is the sum of the individual NSN EBOs. In the example given by Matrix 7, NSN 33 (with a QPA of 4) contributes a large portion of the total EBO.

The "SL" value of 24 similarly shows that the initial stock quantity (which is zero in Matrix 1) needs a lot of augmentation. Because the QPA = 4, however, it does not drive the average NMCS much above 5. The expected NMCS for this part alone would be 3.8 (15.34 ÷ 4).

This matrix contains the most important and useful data for assessing a stock position in toto and part by part. The reader should assure that he has a thorough grasp of the portrayed information.

The average NMCS (100% Cann) and expected backorders are overall indicators of the stock option. A glance at the "SL" column gives a quick indication of which parts are the main contributors to "holes" or NMCS and how much stock is needed to remedy it. If necessary, one can track down through the preceding matrices the various demand and repair characteristics that cause the problem.

We will return to this type display after a short side excursion.

EBOs and Distribution of Holes
(Matrix 8)

The expected NMCS (100% cann) is a useful statistic for describing the condition of the fleet under an extreme cannibalization policy. Given that policy, it distinguishes fairly well between a poor stock position and a good one. It does not, however, tell us very much about the total number of holes or how many holes had to be moved (by canning) to collect them into the smallest number of aircraft displayed by NMCS (100% cann). Neither this model nor Dyna-METRIC tells us how many canns had to be done, but they can tell us about the distribution of holes after 100% canning.

The expected number of holes is the same as the sum of the expected backorders. Summaries of the EBOs for each NSN, each stock option and each time period are shown at the end of each page and at the end of the data annex in the same form as the stock option matrix. A rank-ordered list by EBOs is also a user option.

The distribution of the holes from all NSNs can be calculated as set forth in Appendix C. In addition to Matrix 8, a few examples are shown at the end of the data annex. The distribution of holes characteristically has a wide spread which means that considerable variation from the expected value is likely.

By comparing the expected number of holes and the expected NMCS (100%), one gets an idea of how many holes have to be shoved into the number of NMCS (100%) aircraft, on the average, by cannibalization. (The stock option leading to Matrix 7 gives about 8.4 holes in each of the 5+ NMCS aircraft.) It is at best a gross indication and does not permit an estimate of, for instance, "daily canns." The problem of estimating "daily canns" under various canning policies is a large subject in itself and will be treated at considerable length in another paper of this series.

If time-dependent pipeline models gave us a time history of a single flying unit, we could say more about the canning process. They do not do that, however, except under conditions that are not of much interest. Since that is the case, the reader may well ask: "What do these statistics describe, then, if not the time-track of a single unit?"

A simple answer is the following: To interpret Dyna-METRIC type statistics, we imagine a collection of identical flying units each following its own time track. They are identical insofar as they all are driven by the same assumptions and statistical distributions; they differ in the effect the randomness of events (demands, repairs, etc.) has on each. The statistics are calculated across the ensemble of units at a particular point in time, not for an individual unit across a span of time. In short, the statistics describe the ensemble, not the time behavior of a single specific unit. For many questions dealing with specific flying unit behavior from one day to the next, this theory falls short of what is needed. Offsetting those "disappointments" are the many useful planning applications of average behavior.

Peacetime Stock Options
(Matrices 7 - 12)

Matrices 7 and 9 through 13 show the details of performance for stock options 1 through 6. (The reader should study them before continuing.)

The major indicators are extracted and shown in Table 1 below. With the exception of option 4, each indicates an overall improvement in performance over its predecessor. Option 5 or 6 obviously would be great in peacetime, but they are really based on stock levels which are appropriate to the sums of WRM and POS levels and should produce good peacetime performance.

Table 1

Status on Day 0 (Peacetime)

Stock Option	Average NMCS	Expected Backorders
1	5.35	44.98
2	2.29	8.69
3	1.00	1.56
4	2.95	10.42
5	0.15	0.15
6	0.01	0.01

For real data, we could dwell at length on the differences between options. Here, where our interest is in illustrating content, we give them only a little attention. (See Matrix 1 for stock option vectors.)

- Option 2 effects a general improvement by augmenting some of the worst offender NSNs. (Compare SL columns.)

- Option 3 adds still more stock so that shortages, where they occur, are generally ones and twos spread across all NSNs. (ΣSL_i = 33 needed items.)

- Option 4 (ΣSL_i = 43) leaves the shortages in a relatively few NSNs (note particularly NSN 33) to make the well recognized point that the concentration of shortages is as important as the total number.

- Options 5 and 6 are more properly wartime stock levels than peacetime ones as will be seen in the next section, but they obviously would give very good performance if all that stock were made available in peacetime.

Condition of the Fleet During the Big Surge

(Matrices 14 - 20)

The preceding data were all computed from $\lambda_i(0)$ and, therefore, pertain on day zero, or to "peacetime operations." Of course, the computer programs permit us to look at any day and any stock option we wish, so we now turn to day 10, the end of the major surge effort. At day 10 the supply position will be about as bad as it will get.

Matrices 14 through 20 portray the performance of the various stock options. (The reader should look at them before continuing the text.) Summary data are shown in Table 2.

Table 2

Status on Day 10
(After 10 days of 3.0 surge)

Stock Option	Average NMCS	Expected Backorders
1	16.31	214.19
2	15.60	142.05
3	19.74	130.81
4	9.75	76.62
5	5.40	20.41
6	7.86	18.15

The peacetime options (1, 2, or 3) are probably not feasible. Indeed, the model operates to produce demands as if the flying program were carried out, but it may not be possible to fly 144 sorties per day (48 x 3.0) with only 28 to 32 aircraft non-NMCS.

Option 4 performed poorer than option 3 during peacetime but significantly outperforms it here. (Surprise?) By looking at the appropriate matrices (17 and 18), we see that for a surge, NSN 7 was grossly understocked (SL = 57) in option 3, much less so (SL = 24) in option 4. In peacetime, NSN 7 was available in reasonably adequate quantities for both options (Matrix 10, SL=2 for option 3; Matrix 11, SL=0 for option 4; the actual stock levels are: 23 for option 3 and 56 for option 4). Here again, we sneakily buried a hook in the options to emphasize that a single NSN can cause disastrous drops in performance. This clearly demonstrates that "fill rate" (fraction of total parts available out of total required) is a dangerous indicator of WRM status.

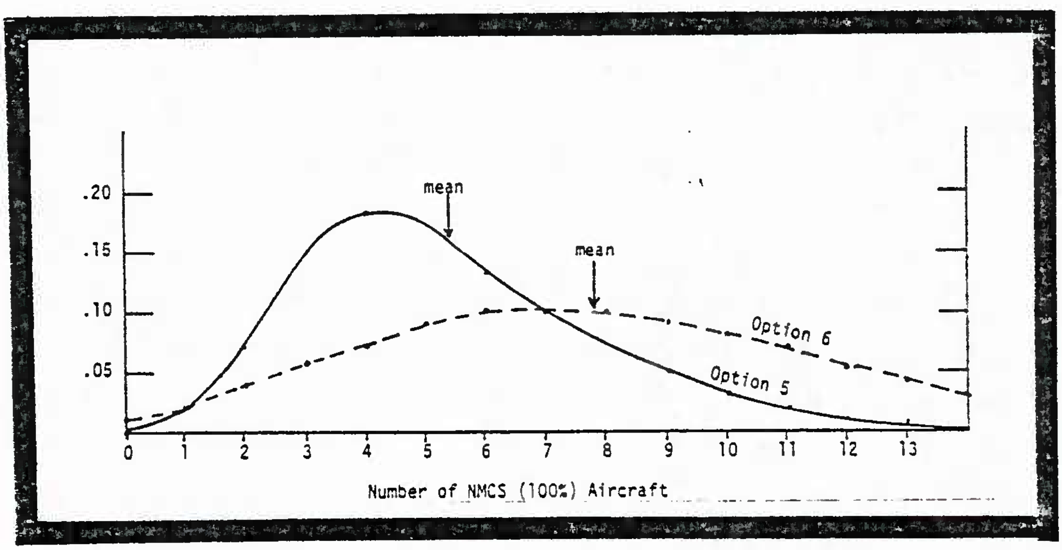
Option 5 looks better than option 6 on day 10 (but neither looks good). Since the expected backorders are higher for #5 than for #6, we suspect that #6 has less evenly distributed shortages. From Matrices 19 and 20, the suspicion is confirmed: NSN #7 is again the culprit.

We note also in option 5, however, that there are two critical items: NSN 7 and NSN 33. The second one (or the first, if you prefer, since they are somewhat symmetrical) tends to mask the effect of the other when looking at the NMCS (100% cann) indicator. Each of the NMCS group for this option would tend to have two holes rather than just one. If we wanted to improve option 5, we would need to add stock in both bins.

Although it is not clear which option is "better" than the other, it is becoming clear that they are different: #6 has one dominant critical part while #5 has two dominant critical parts. Looking at the full NMCS (100% cann) distribution permits us to emphasize that we are using "expectations" to describe something that in the actual event may be quite different; it also shows how the "two-versus-one" aspect of criticality (and its extension to cases of "many-versus-few") influences what we are apt to see in a real-world trial.

Both of the frequency functions in Figure 2 are characterized by rather significant spreads. (A statistician would say that the standard deviations are relatively large.) In both cases, were we to have a real-world trial, it would be quite likely we would observe some value other than the expectation (i.e., mean). Although option 6 would, on the

Figure 3 Distributions of NMCS Aircraft For Stock Options 5 and 6
(Surge Day 10)



average, perform poorer than option 5, it could turn out better in any one trial! Option 5 is more "peaked" and less "spread out," due to the combined effects of the two critical parts. The more parts with nearly the same criticality, the sharper the peak.

Condition of the Fleet During Recovery

(Matrices 22 - 27)

On the 11th day, the sortie rate was dropped to 1.00 and kept there thereafter. See Table 3.

Table 3

Status on Days 0, 10, and 30

	Day 0		Day 10		Day 30	
Stock Option	NMCS* A/C	EBO	NMCS* A/C	EBO	NMCS* A/C	EBO
1	5.35	44.98	16.31	214.19	13.35	173.12
2	2.29	8.69	15.60	142.05	10.12	101.93
3	1.00	1.56	19.74	130.81	8.58	81.81
4	2.95	10.42	9.75	76.62	9.64	73.31
5	0.15	0.15	5.40	20.41	4.44	17.37
6	0.01	0.01	7.86	18.15	1.94	3.60

* 100% cannibalization

When we look at day 30, we see very different recoveries for the various options. Option #5 does not recover as well as #6, for instance. Looking at Matrix 26, we see this is due to NSN 33 (SL = 21) which was masked at the 10-day mark by NSN 7. NSN 7 was also the dominant reason for option 6's poor performance at the 10-day mark. On reflection, then,

it is apparent that NSN 7 recovers faster than NSN 33. Looking back at Matrices 2, 3, and 4 we find the data shown in Table 4 below:

Table 4

Probabilities of Repair and Repair Times

	Prob of Base Repair	Repair Time	Prob of CIRF Repair	Repair Time	Prob of Depot Repair	Repair Time
NSN 7	.85	8 days	0	-	.15	31.5 days
NSN 33	.27	5 days	0	-	.73	31.5 days

Thus, NSN 33, most of which are repaired at the depot, has not begun to recover by day 30; but NSN 7 with most of its repair accomplished at the base recovers rather quickly.

Matrix 5 which shows the $\lambda(t)$ vectors for the whole pipeline as a function of time shows the behavior quite clearly. The expected number of NSN 33 grows quickly during the 10-day surge to 36.8, then slowly continues to increase to 40.1 on day 30. The expected number of NSN 7 in repair peaks on day 10 at 61.9, then drops to a low of 31.7 on day 18 and (due to the fraction that goes to the depot) slowly climbs to 33.4 on day 30.

We would, of course, expect a fast recovery for both parts during the period between 30 and 40 days because the large quantities that went to the depot during the initial 10 days would begin to return.

Influence of Different Forms of the Repair Function

The matrices discussed in the preceding part of this note are based on constant repair functions with constant shipping times. The parameters

were set equal to the average time-of-repair observed for each NSN and for the repair pipeline being considered. Alternatively, we could have used exponential distributions for both the repair function and shipping time, a form built into the Rand Dyna-METRIC programs.

Under steady-state conditions, Palm's theorem says that the form of the repair distribution is immaterial, only the mean value is influential. But such is definitely not true for transient conditions; there, the form of the distribution can have an impact on the stock performance under some conditions. In a later report, we will explore what influence the different repair functions have, for that is a moderately complex story in itself and is better treated with real-world data inputs.

IV. Summary

The matrix presentations of supply performance which we have described herein should make clear, on the one hand, just how complicated the overall logistics problem really is! On the other hand, the presentations also make it evident that we are getting into an ever-better position to grapple with those complexities in practical and useful ways:

> *A moderately complex stochastic model is used to "crunch" a ton of data, on hundreds of LRUs, over an extended period of time spanning various surges in sortie production and time-dependent stock levels, incorporating several repair pipelines which may use different kinds of functions to describe the repair process -- and then the outputs are presented in ways interpretable in terms of the unit's overall ability to produce combat sorties.*

Throughout this paper, we have hinted at possible uses of the model. There are so many we will not try to catalog them here. We have some very clear notions about immediate applications:

- A thorough factoring of the F-16 spares position both in peacetime and wartime
- An offshoot based on these underlying notions which will explore in detail several cannibalization policies in our F-16 unit
- An overall assessment of our F-4 wartime capabilities
- Explorations (along the way) of the sensitivity of the model results to the uncertainties imbedded in the real-world data base

While building the data base is painfully slow, the computer crunches it quickly. To produce the matrix outputs in the data annex took about six minutes of computer core time. Moreover, the basic $\lambda(t)$ matrix that embodies much of the calculational time needs to be computed only once for a given set of demand and repair functions. It may then be stored on disk and used for all stock option explorations. Since the fundamental repair data base is only occasionally changed, this is a very fast computational model. It consumes a lot of paper, but it doesn't use much computer time.

REFERENCES

1) R. J. Hillestad and M. J. Carrillo, "Models and Techniques for Recoverable Item Stockage When Demand and the Repair Process Are Non-stationary--Part I: Performance Measurement," N-1482-AF, The Rand Corporation, Santa Monica, California, May 1980.

2) G. B. Crawford, "WRSK/BLSS Analysis, I. Generalization of Palm's Theorem," HQ PACAF/Operations Analysis, Hickam AFB, Hawaii, March 1977.

3) W. Feller, An Introduction to Probability Theory and Its Applications, Vol. 1, John Wiley & Sons, New York, 1950.

Appendix A

"The Importance of the "Poisson Property"

It is indeed fortunate that we can usually represent spare part "demands" by a Poisson process, one in which the demands per unit time vary stochastically according to a Poisson distribution with expectation m. The Poisson distribution has a marvelous property: It reproduces itself. The property is demonstrated in most statistic text books and frequently expressed as a theorem:

Theorem

If S_n is a sum of n independent random Poisson variates, x_i, having means, m_i, then S_n is a Poisson variate with expectation, Σm_i.

Dr. Gordon Crawford's original proof of the time-dependent Palm's theorem which we quote in Appendix B calls on that theorem several times.

The Poisson property also provides the justification for "factoring" a many-loop repair process into individual loop processes for calculations, then recombining them at the end. To illustrate:

> Consider the Poisson demands for part i on day k characterized by the expected value $m_i(k)$. Imagine now an additional random process, independent of the demands themselves, which will operate on each realized demand so as to put it in one of three bins labeled A, B, and C according to probabilities P_A, P_B, and P_C. Under this process, the contents of A, B, and C on day k will be Poisson variates with means $m(k)P_A$, $m(k)P_B$, and $m(k)P_C$. Now, we complete the picture by letting A be the base repair cycle, B the CIRF repair cycle, and C the depot repair cycle. Although each has a different repair function and demand function, we know from the time-dependent Palm's theorem that the contents on day k are Poisson variates and we know how to compute the expected value. The contents of each repair path are, moreover, statistically independent so the Poisson property tells us that the

totality of parts in the repair process is a Poisson variate whose expectation is the sum of the expectations for each repair path.

When we can meet the requirements of independent Poisson demands, we are in good shape. Indeed, the model at hand is very powerful: We do simple algebraic calculations on expected values and end up knowing all of the distributional behavior as well!

But there are model extensions we would like to explore which violate the all-important assumption of statistical independence. First and probably foremost is the "indenture relationship" between LRUs and SRUs. If the repair function of the LRUs is made to depend upon the stock of SRUs which in turn is a function of past demands, we clearly have "independence" problems and our useful appeal to the Poisson property is no longer valid. Similarly, if the repair function is permitted to depend on the demands themselves (as would occur if the CIRF repair time for each part becomes longer when demands increase), then we lose independence and our simple model is no longer valid. To retain it, we must pretend that the CIRF expands its capability at just the right rate to guarantee repair function is not dependent on the number of parts in repair. This can be met by having a repair process with an infinite number of servers, each of which has the postulated repair probability function. That is not apt to be the case.

Appendix B

Evaluating the Expected Number in Repair

In the main body of the report we cited the RAND formulation of the time-dependent Palm's theorem which is in a very general form, and immediately thereafter wrote down a discrete form with a sum replacing the integral. It is instructive to start from scratch in this appendix by reproducing a portion of a previous OA report (ref 2) written by Dr. Gordon Crawford who set down one of the earlier proofs of the extension to Palm's theorem. We have changed his labeling convention to that used in this paper.

> To fix ideas and conventions, let us assume that if an item has a repair/replace time of, say, two days, and if it fails on day i, then it is out of service on day i + 1 and comes back in service on day i + 2. In this case, we say a "due-in" existed on days i and i + 1. (Note: This convention assumes all accounting actions occur at the end of each day.)
>
> To examine the question of how many parts are needed to support a sortie surge, we formulate a model which allows a calculation of the distribution of the number of due-ins on each day of a war.
>
> We begin by assuming that the number of demands on day i is a Poisson random variable (r.v.) with mean m(i), m(i) $\geq$ 0, i = 1, 2,.... It is this feature of a changing mean demand rate that characterizes the increasing tempo of operations associated with the initiation of combat.
>
> Suppose repair times are constant and equal to some integer n_0, $n_0 \geq 1$. If $n_0 = 1$ then the number of due-ins on day k is the number of demands on day k, that is, the number of due-ins is a Poisson random variable (r.v.) with mean m(k). If $n_0 = 2$ the number of due-ins on day k is a Poisson r.v. with mean m(k-1) + m(k).

More generally, but by the same reasoning, if repair time is constant and equals n_0, the number of due-ins on day k is a Poisson r.v. with mean*

$$\text{i)}\qquad \sum_{j=k-n_0+1}^{k} m(j)$$

If repair times are not fixed, as in the preceding, but more generally are integer-valued random variables and if they are independent identically distributed and independent of demands and if they assume the values n_i with probability p_i, i = 1, 2, ..., then the number of demands on day j with repair time = n_i is a Poisson r.v. with mean $p_i m(j)$. Moreover, this r.v. is independent of the number of demands on day j with repair time = n_m, $m \neq i$ (which in turn is a Poisson r.v. with mean $p_m m(j)$).

It follows from i) above that the number of due-ins on day k with repair time n_i is the sum of n_i independent Poisson r.v.s and hence is a Poisson r.v. with mean

$$\sum_{j=k-n_i+1}^{k} p_i m(j)$$

By virtue of the above-mentioned independence, we have shown that the total number of due-ins on day k is a Poisson r.v. with mean

$$\text{ii)}\qquad PI(k) = \sum_{i=1}^{\infty} p_i \sum_{j=k-n_i+1}^{k} m(j)$$

This result is a special case of a more general form of Palm's Theorem:

<u>Palm's Theorem</u>

If M is a non-negative function such that the number of demands in the time interval $(t_1, t_2]$ is a Poisson random variable with the mean $M(t_2) - M(t_1)$, and if repair times are independent identically distributed random variables with arbitrary distribution function F,

$$F(x) = \Pr[\text{repair time} \leq x], \qquad 0 \leq x < +\infty,$$

*This result and some of its ramifications were suggested to us by T. Lippiatt of the Rand Corporation.

and if repair times are independent of demands, then the number of due-ins at time t is a Poisson random variable with mean PI(t), where

iii) $$PI(t) = \int_0^\infty [M(t) - M(t-x)]dF(x).$$

The integral iii) is the Lebesque-Stieltjes integral with respect to the measure induced by F. Monotonicity of M (and hence integrability) follows from the understanding that the mean of a Poisson r.v. is non-negative.

The proof of the generalized Palm's theorem, not reproduced here, then follows in Dr. Crawford's paper. We are satisfied with his equation (ii) which is shown to be equivalent to our equation (2) (page 7) as follows:

$$\begin{aligned}
PI(k) = \;& p_1 [m(k)] \\
+\;& p_2 [m(k-1) + m(k)] \\
+\;& p_3 [m(k-2) + m(k-1) + m(k)] \\
+\;& \text{etc.}
\end{aligned}$$

$$\begin{aligned}
= \;& m(k)\ [p_1 + p_2 + p_3 + \ldots] \\
+\;& m(k-1)[\phantom{p_1 +{}} p_2 + p_3 + \ldots] \\
+\;& m(k-2)[\phantom{p_1 + p_2 +{}} p_3 + \ldots] \\
+\;& \text{etc.}
\end{aligned}$$

$$\begin{aligned}
= \;& m(k)\ [\ 1\ ] \\
+\;& m(k-1)[1-p_1\ ] \\
+\;& m(k-2)[1-p_1-p_2] \\
+\;& \text{etc.}
\end{aligned}$$

$$\begin{aligned}
= \;& m(k)\ \ \overline{F}(k,\ k) \\
+\;& m(k-1)\ \overline{F}(k-1,\ k) \\
+\;& m(k-2)\ \overline{F}(k-2,\ k) \\
+\;& \text{etc.}
\end{aligned}$$

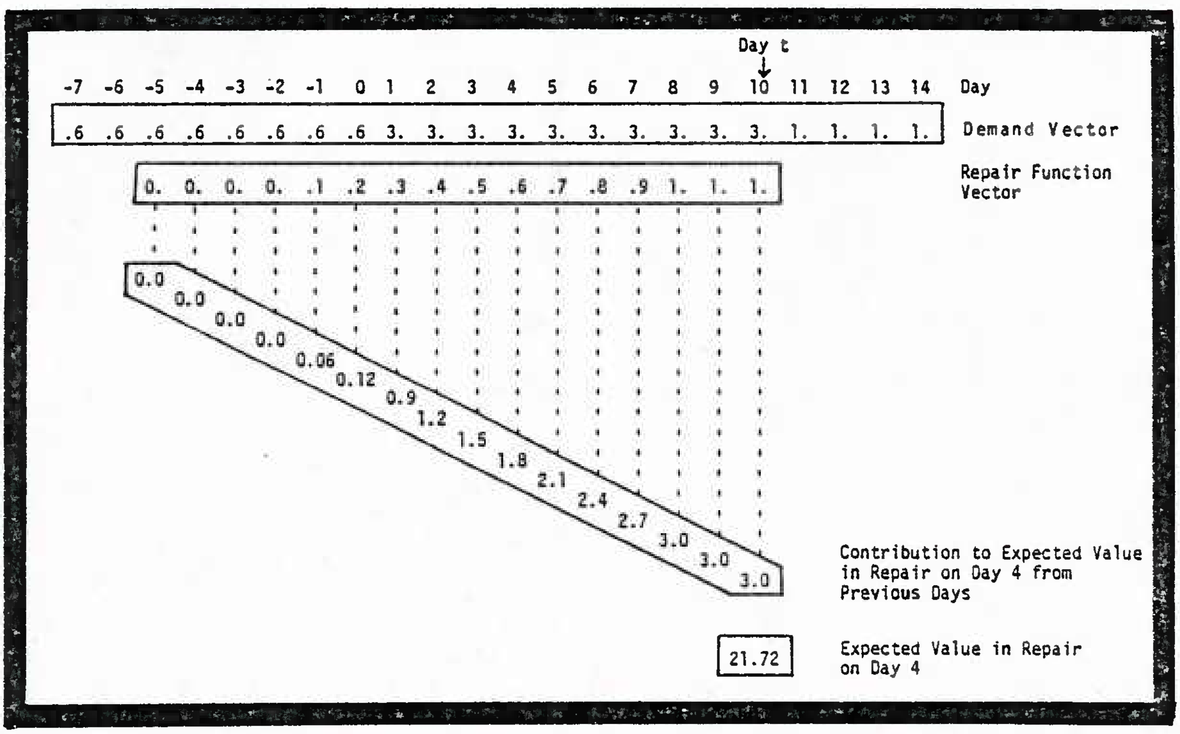
where $\overline{F} = 1\text{-}F$

$$= \sum_{s=0}^{s=k} m(s)\ \overline{F}(s,k) = \sum_{s=0}^{s=t} m(s)\ \overline{F}(s,t)$$

Suppose today is day k. The formula simply says that the number in repair at the end of today are those that arrive today, plus the fraction of those that arrived yesterday which have not been repaired, plus the fraction of those that arrived day before yesterday which have not been repaired, etc., on back until either we're at the beginning of the process or we're sure that demands for that day and before have certainly been repaired.

A graphical illustration follows:

Consider the figure below showing a demand vector and a repair function vector, $\overline{F}(s,t)$, so called "vectors" because they are defined by a list of numbers.



The sortie rate vector (not shown) jumps to a surge level on day 1 after a long initial period of steady-state training. On day 11, the sortie rate drops, but not as low as in peacetime. The mean demand vector m(t) for the part in question is proportional to this sortie rate vector, each part having its own characteristic "demands per flying hour."

The repair function vector in this illustration is a simple one: There is a 3-day period (today, yesterday, and the day before) during which no parts are repaired (perhaps representing fixed transportation and processing times), a 10-day period during which there is a uniform probability of repair, and a still earlier period from which all demands will have been repaired.

To find the "expected value in repair on day t" place the repair vector index on day t of the demand vector and sum the products. As t is changed, so will the "expected value in repair" in a way which should now be obvious. It should also be clear "how long a transient condition persists" or "what steady-state is reached", etc. The transient condition persists as long as the repair function spans any non-constant portion of the demand intensity function.

There are no restrictions on admissible functions other than "statistical independence" and the repair vector satisfying the obvious conditions of probability distributions. We are otherwise free to choose any which suit our fancy. Those who may have pondered why Palm's theorem for the steady-state condition depends only on the mean repair time and not on the form of the repair distribution are now in a position to see clearly why it is true for the steady state and not true for time-dependent demands.

Appendix C

A Note on Convolutions, Distributions of Holes and Expectations of Parts in Repair

Convolutions (Following Feller, ref 3, p 214)

Let X and Y be non-negative independent integral-value random variables (r.v.) with the probability distributions $Pr\{X=j\} = \{a_j\}$ and $Pr\{Y = j\}=\{b_j\}$. The event $(X = j, Y = k)$ has probability $a_j b_k$. The sum $S = x + y$ is a new r.v., and the event $S = r$ is the union of the events:

$$(X = 0, Y = r), (X = 1, Y = r-1), (X = 2, Y = r-2), \ldots, (X = r, Y = 0).$$

These events are mutually exclusive, and therefore the distribution

$C_r = Pr\{S=r\}$ is given by

$$C_r = a_0 b_r + a_1 b_{r-1} + a_2 b_{r-2} + \ldots + a_{r-1} b_1 + a_r b_0.$$

The above operation occurs frequently and is given a special name -- "convolution of the sequence a with the sequence b " -- and a special notation.

$$\{c_k\} = \{a_k\}*\{b_k\}$$

When the sequences $\{a_k\}$ and $\{b_k\}$ are probability distributions the sequence $\{c_k\}$ is a probability distribution.

Feller shows by use of generating functions that successive convolutions of sequences, for example:

$$\{e_k\}= \{a_k\}*\{b_k\}*\{c_k\}*\{d_k\} ,$$

enjoy associative and commutative properties--they can be formed in any order and in any groups.

From the definition, it follows that successive convolutions of sequences which represent probability distributions of r.v.'s yields the probability distribution of the sum of the r.v.'s.

It should be noted that the sequences do not have to be of equal length and that the number of terms in a sequence resulting from a convolution is the sum of the numbers of terms in each of the convolution sequences.

Note also (for computer programming purposes) that a convolution sequence can be visualized from the following matrix:

$$
\begin{array}{c|cccccc}
 & a_0 & a_1 & a_2 & \cdots & a_{k-1} & a_k \\
\hline
b_0 & a_0b_0 & a_1b_0 & a_2b_0 & \cdots & a_{k-1}b_0 & a_kb_0 \\
b_1 & a_0b_1 & a_1b_1 & a_2b_1 & \cdots & a_{k-1}b_1 & a_kb_1 \\
b_2 & a_0b_2 & a_1b_2 & a_2b_2 & \cdots & a_{k-1}b_2 & a_kb_2 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
b_n & a_0b_n & a_1b_n & a_2b_n & \cdots & a_{k-1}b_n & a_kb_n
\end{array}
$$

Each element of the sequence is generated by the diagonal sums with k equal to the sum of the subscripts. A computer program to perform convolutions is one which sums the diagonals and stores them as a labeled sequence.

Distribution of Number of Holes

Let $Pr\{\text{holes } (NSN_i)=j\} = \{h_j(i)\}$, that is $\{h_j(i)\}$ is the sequence that gives the probability distribution of holes for the i^{th} NSN.

From the preceding, the distribution of holes covered by two NSNs, say m and n, is given by:

$$\{h_j(m)\}*\{h_j(n)\} \quad .$$

The distribution of total holes caued by all NSNs is

$$\{h_j(1)\}*\{h_j(2)\}*\{h_j(3)\}*\ldots *\{h_j(N)\}$$

where N is the last stock number.

Thus by taking successive convolutions of the distributions of holes for each NSN with the preceding convolutions, the distribution of holes arising from all NSNs is generated. The process is easily carried out at the same time the matrix displays are calculated.

Expectations of Parts in Repair

In Appendix B, we derived the expected number of part i in repair on day t given the demand functions m(s) and repair function $\overline{F}(s,t)$:

$$\lambda_i(t) = \sum_{s=0}^{s=t} m(s)\, \overline{F}(s,t).$$

In the light of the previous discussion, it will be recognized that the sequence $\{\lambda_i(t)\}$ is given by the convolution of $\{m(s)\}$ and $\{\overline{F}(s,t)\}$.

DISTRIBUTION

HQ USAF/SAG 1
LE 1
Wash DC 20330

HQ AFLC/XRS 1
LORA 1
MMMA 1
Wright-Patterson AFB OH 45433

HQ MAC/LG 1
Scott AFB IL 62225

HQ PACAF/OA 8
Hickam AFB HI 96853

HQ SAC/NR 1
Offutt AFB NE 68113

HQ TAC/LG 1
Langley AFB VA 23665

HQ USAFE/DOA 1
LG 1
APO New York 09012

USAFA/DFMS (Maj R. Rue) 1
USAF Academy CO 80840

AFHRL 1
Human Factors Div (Mr F. Maher)
Wright-Patterson AFB OH 45433

AFIT/LDEJ 1
LSM 1
Wright-Patterson AFB OH 45433

AFLMC/LG 1
Gunter AFS AL 36114

Dynamics Research Corp 1
Attn: Mr Limpert
60 Concord Street
Wilmington MA 01887

The Rand Corporation 2
Attn: Dr G. Crawford
1700 Main Street
Santa Monica CA 90406

Logistics Management Institute 1
6400 Goldsboro Road
Bethesda MD 20817-5886

DTIC 12
Cameron Station
Alexandria VA 22314

DLSIE/ALMC 1
US Army Logistics Mgt Ctr
Ft Lee VA 23801

Naval Postgraduate School
Adm Sciences Dept, Code 54 1
Library, Code 0142 1
Monterey CA 93940

AU AUL/LD 1
Maxwell AFB AL 36112

HQ ATC/LG 1
Randolph AFB TX 78150

HQ AAC/LGSM 1
Elmendorf AFB AK 99506